

Active Learning of Mathematics

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Published in:

Benton, N & R.(eds.) (2001) *Te Rito o te Maturanga: Experiential Learning for the Third Millenium*. Vol. 2, 159-168, Auckland: James Henare Maori Research Centre for the International Consortium for Experiential Learning.
(ISBN 0-9582366-1-5)

Abstract

In a traditional learning setting the teacher often transmits the mathematics curriculum according to what is said in a textbook and in a strict and rather learner-passive manner. During the last decades research has pointed out controversies in this approach to teaching and learning mathematics. In the present article the author discusses some of these controversies and introduces a different approach to teaching and learning mathematics, the active learning approach. The active learning approach was used in a workshop at the ICEL2000 called "Active learning of mathematics". The structure of this workshop is presented in the article. *Key words:* traditional teaching of mathematics, active learning, learning culture

*The range of what we think and do
is limited by what we fail to notice.
And because we fail to notice
that we fail to notice,
there is little that we can do
to change, until we notice
how our failing to notice
shapes our thoughts and deeds.*

R. D. Laing

Introduction

Mathematics is a subject that often divides people into two groups. There are those who say that they are skilled at mathematics and who loved the mathematics lessons at school and there are those who say that they hated it and "blocked" at the first drop of a symbol (Ernest, 1996; Skemp, 1986). People in the latter group often go on telling horror stories about past encounters with mathematics in school. The devastating feelings of not understanding the "name of the game" and lacking a "mathematical mind" often run all through the stories. What is it about mathematics at school that creates such strong opinions for or against? This is a very legitimate question especially when you consider that school-mathematics often serves as an

instrument of selection for further studies and, hence, the teaching of mathematics in school may have a crucial influence on the personal and professional life of the learner. The special role of mathematics is accentuated by the society since mathematics is commonly used to legitimate policies and decisions. According to Lerman (1993) and others (i.e. Ernest, 1991, 1996; Fennema & Nelson, 1997; Keitel, 1989; Thompson, 1989) this places school-mathematics and teachers of mathematics in a unique position.

Many results of empirical research and discussions about mathematics teaching during the last two decades, support the view that an active and social approach to the teaching and learning of mathematics might be a way to prevent undesirable and negative effects (i.e. Keitel, 1989; Ernest, 1991; Seeger et.al., 1998). Among other factors, such a pedagogical approach puts forward the necessity of social construction of mathematical meaning and the role of the teacher as facilitator in this construction process. It includes a view of the learner as active problem-solvers working individually and in small groups to make connections between multiple forms of representations of mathematical concepts, i.e. spoken symbols, written symbols, concrete models, graphics and real-world situations. (Black & Atkins, 1996; Dance, 1997; Ernest, 1991; Robertson et. al., 1994; Wood, Cobb & Yackel, 1991)

Evidently, an alternative approach to teaching and learning includes changes in the pedagogical practices of the mathematics classroom. According to Grouws & Cebulla (2000) mathematics instruction should be focused on *meaningful development of important mathematical ideas* and *highlight the mathematical meanings of these ideas*. This includes how the idea, concept or skill is connected in multiple ways to other mathematical ideas and forms of representations in a logically consistent and sensible manner. Further, mathematics instruction should give the learners an opportunity to *discover* new knowledge and to *practice* what they have learned as well as to *connect* mathematics to other subjects and to the world outside school. The teaching methods that are implemented in mathematics instruction should incorporate, and also make explicit, intuitive ways of finding solutions, combined with opportunities for *verbal interaction* in small groups or in whole-class discussions. Hence, the suggestions put forward by Grouws & Cebulla reflect a view of the learner as an active participant in the pedagogical processes in the classroom¹.

In the following the "traditional" approach to teaching and learning mathematics is first discussed followed by an introduction to a so-called "active learning" approach. I am aware of the inherent risk of oversimplifying when using an "either-or" philosophy and setting "traditional" education against some other pedagogical practice that is supposed to be more appropriate. Dewey used the same kind of approach when he discussed "progressive education" as the opposite to "traditional education", but he also expressed a pragmatic realism in the statement that "extremes cannot be acted upon ... when it comes to practical matters circumstances compel us to compromise". (Dewey, 1938, 1997, 17) After working as a mathematics teacher for many years, my own experience tells me that Dewey is right.

¹ I define "classroom" as the place where mathematics instruction takes place regardless of its physical location.

Nevertheless I think there might be a point in starting from an extreme here, in order to make the reader aware of where many mathematics teachers and learners start when they begin their journey of change.

The traditional approach to teaching and learning mathematics

In the following brief account I want to give an idea of what constitutes "traditional mathematics education"; the kind of teaching and learning many a mathematics teacher is trying to leave behind. (Black & Atkin, 1996; Fennema & Nelson, 1997) The picture I present is especially true for the junior and senior secondary school level. (Bodin & Capponi, 1996)

According to Lerman (1993) the teacher, the teaching methods and the textbook can be considered as the most important factors influencing student's attitudes to school mathematics. This conclusion is strongly supported in research reports and discussions about mathematics education.

In a survey that included 132 mathematics teachers at junior secondary level² in 42 Swedish-speaking schools in Finland, 90% of the teachers told that they often use the textbook, mostly in conjunction with exercise books. The survey also showed that teacher-centred teaching (often used by 85% of the teachers) and individual seatwork (often used by 72% of the teachers) were the main teaching methods used in mathematics. (Røj-Lindberg, 1999) Less than half of the teachers, 42%, noted teacher-led whole-class discussions as a method often used, while only 25 % often used small-group problem-solving. Thus, a majority of the mathematics lessons in the Swedish-speaking³ junior secondary schools in Finland, at the time of the survey, seemed to be dominated by learner's individual seatwork and by teacher-centred and transmission methods of teaching. Results from the TIMSS-R⁴ study showed that nearly all mathematics teachers in Finland prefer textbook teaching at the 7th grade regardless of language of delivery. (Törnroos, 2001) An on-going study, where pupils in a junior secondary school were asked to describe what is going on during the lessons, confirms the learner-passive nature of the traditional mathematics instruction. (Røj-Lindberg, 2001) As no similar national survey has been done, I cannot draw any firm conclusions to the mathematics teaching in general at the junior secondary level in Finland. Results presented by Kupari (1999) do indicate, though, that mathematics teaching in grade 9 is often structured according to traditional teaching methods.

Bodin & Capponi (1996) discuss mathematics teaching at the junior secondary level. When comparing their own research with indications from other available comparative studies they find similarities between current practice and the picture

² Pupils in the age of 13 to 16 years

³ Finland has two national languages, Swedish and Finnish, 6% of the population are Swedish-speakers, 94% Finnish-speakers.

⁴ TIMSS-R = The Third International Mathematics and Science Study - Repeat. This study involved teachers and pupils at the 7th grade in 38 countries in year 1999.

drawn at the conclusion of the SIMS⁵ (Second International Mathematics Study) (citing Robitaille, p. 574):

The predominant mode of instruction is teacher-dominated. The class is taught as a whole, and all students are expected to cover the same amount of material, in the same way, and at more or less the same pace. The approach may be characterized as business-like and fairly highly structured. Discipline is important, but these experienced teachers have little difficulty maintaining what they believe to be an appropriate level of control. Teachers generally do not attach much importance to the development of better attitudes or self-concepts among their students, nor do they believe that instruction should be modified to take account of the needs, interests, and abilities of their students.

The learner's mathematical ability and the hierarchy of the subject appear to be central to the way many mathematics teachers think about mathematics learning, and their organisational and pedagogical practice. (Ruthven, 1987) Many teachers seem to mainly focus on explaining algorithms and rules to learners, who are supposed to implement these when they solve tasks that appear in the textbook. Learners' individual work with rules and symbols is a predominant teaching method. The individual work is often differentiated according the task difficulty.

To know and be able to implement the right steps to reach the right answers is a central goal. Bodin & Capponi conclude that "the traditional course described corresponds to a very widespread practice" (1996, p. 585). According to Ernest (1996) the traditional mathematics instruction transmits a view of mathematics as straightforward, logical, absolute and, in most cases, disconnected from reality and independent of the persons of both learner and teacher. The role of the learner is to listen carefully to the teacher and to learn and understand by doing lots of exercises, one after the other, and preferably by working alone in silence. The role of the learner is also to memorise the facts and rules that are sanctioned by the teacher and to implement them. The teacher quoted in the article *When Good Teaching Leads to Bad Results*, expresses this division of roles between teacher and learner, when she urges the students: "*you will have to know all your constructions cold so you do not spend a lot of time thinking about them*". (Schoenfeld, 1988)

To sum up, the learning process within the traditional approach is focused on finding the right answers to problems in the textbook. Hence, if the learner fails it might be due to a teacher who is poor at telling and explaining. The reason for failure is also often seen to be inherent in the learner as lack of mathematical intelligence or lack of self-discipline or a combination of both (Ernest, 1991; Rudduck et al., 1996). Thus, critically speaking, a learner is likely to experience success in traditional school-mathematics if she is lucky both to be talented and to have a teacher who is gifted in explaining the algorithms, rules and models, and in keeping up law and order in the classroom. Also, this learner will most certainly agree with the system of expectations held by the learners and the teacher, in other words, all the rules, mostly

⁵ SIMS = Second International Mathematics Study. SIMS was conducted during the 1981-1982 school year. It had 20 countries collecting data on students in grades with the most 13-year-olds and those students taking mathematics in their final year of secondary school as a substantial part of an academic program.

implicit, which constitute the culture of the classroom. (Brousseau, 1990; Seeger et al., 1998)

In general, teachers of mathematics set up ambitious goals for their work and would like all their learners to love mathematics and, also, to reach a high level of learning and understanding. However, the traditional teaching approach might be among factors that work against these goals. Richard Skemp, a famous mathematician and psychologist, describes the type of learning that often is an effect of the traditional approach as habit learning or rote-memorising. (Skemp, 1986) Moreover, Skemp (1976) has described the type of understanding that is related to habit learning as instrumental understanding. The learner knows separate chunks of mathematical knowledge by heart and applies those "as a pigeon" without deep understanding of the inherent mathematical structure and how the different knowledge chunks are connected. This type of knowledge without deep understanding of the used concepts could be called procedural (Hiebert & Lefevre, 1986; Rittle-Johnson & Siegler, 1998).

Moreover, according to Schoenfeld (1988) the traditional teaching approach is likely to result in beliefs that are detrimental to intelligent learning, which Skemp defines as the opposite to habit learning. Such an evolving belief is, for instance, that a mathematical problem is solvable in just a few minutes. If the learner has not reached the right answer after a few minutes she thinks that she has not understood the mathematics and needs help. The learner does not develop the perseverance that is needed in solving mathematical problems and in mathematical thinking of a higher order. Another effect is that the learner develops an image of herself as a passive learner and consumer of mathematics. The learners end up in believing that "*someone else's mathematics is theirs to memorise and spit back*" (Schoenfeld, 1988; Skemp, 1986).

The goal of the teaching process in mathematics should be to establish a learning culture that promotes intelligent learning and deep understanding of the mathematical concepts. The knowledge of the learned mathematical concepts could in such a case be called conceptual (Hiebert & Lefevre, 1986; Rittle-Johnson & Siegler, 1998). This type of deep understanding of the conceptual structure is by Skemp (1976) described as relational understanding. Tanner and colleagues studied classroom practices in mathematics in twenty schools and found that particularly successful schools emphasized relational rather than instrumental understanding. (Tanner & Jones, 2000, 21)

In short, I think conceptual knowledge and relational understanding is generated and intelligent learning occurs when the learner is given the possibility to actively create rich structures of cognitive connections within and between mathematical concepts. Hence, the teaching procedures and interpersonal relations in the mathematics classroom should provide a framework that enhances the prosperity of such a creative and active learning culture. I will call this type of framework *an active learning approach*.

A different approach to learning and teaching mathematics: the active learning approach.

Within the active learning approach the learner actively constructs her understanding of mathematical concepts in an iterative fashion. In this case the word iterative means that the structure and content of the teaching process is enhancing procedural (being sure of which steps to take) and conceptual (to know the meaning, to know why the steps are taken) knowledge to evolve simultaneously. Hence a more and more complex cognitive network of relationships between different pieces of mathematical information is forming in the mind of the learner. The cognitive network includes knowledge of both procedures and concepts, knowledge that is connected in ways that give the learner both a good intuitive feel for mathematics and a good problem-solving and answer-generating capacity (Hiebert & Lefevre 1986, 9).

An episode from my work with teacher students may illustrate how conceptual knowledge suddenly can be constructed in the mind of the learner out of previously unrelated pieces of knowledge in a supporting social situation. We were discussing multidigit subtraction, the subtraction algorithm and the positional value of each digit and the effects of moving from one position to another in performing the subtraction. We also worked out some algorithms together with the help of concrete objects illustrating the digits in the bigger number. Then one student suddenly said that now he understands why he always writes a small nine on top of the zero as he is using the algorithm when solving problems like $8049 - 1756$! He was a good student and had performed lots of subtractions with good results, but obviously he had never established a deep and conceptual understanding of what he was doing. The subtraction algorithm was just a useful ritual to him that always produced the right answer. He was very aware of the surface features but he had never really understood the meaning behind the manipulation of the symbols.

So what was the key to his sudden insight? As mathematical understanding is an internal state of mind that cannot be viewed directly, you cannot be sure. But I think it was a consequence of the reflective discussion in the group, in combination with the simultaneous use of concrete objects and written symbols to illustrate the algorithm. The discussion helped him become more conscious and aware of the limitations in his understanding of the manipulation of symbols.

In the following I will briefly discuss the active learning approach on the basis of four statements: about the learner, about the teacher, about the learning task and about the interaction in the mathematics classroom.

1. *The learner is the key-person in the active learning process, i.e. mathematics teaching should be learner-centred.*

By using the concept "learner-centred" I want to acknowledge the learner's need to be active mentally, socially and physically. I also take into account that when

school-mathematics is integrated with (messy) real-life applications and when the learner finds the tasks meaningful and interesting, intelligent learning and relational understanding have a bigger chance to evolve. Moreover, a mathematical problem may be mentally challenging, but if the learner is not interested in it, she will not be challenged by it (i.e. Boaler, 1997). We should also consider the advice by Skemp (1986) that practical, oral and mental work can provide the foundation of understanding without which written work makes no sense. This may be illustrated with the metaphor of learning how to ride a bike by just reading about biking and listening to lectures about how the different parts of the bike work in combination with each other. You need to practice biking actively in order to learn!

2. The teacher is an authority because of her knowledge of both mathematics and mathematics learning and her respect for the learners as thinking and socially active individuals.

With this statement I want to stress that a prerequisite to implementing a successful change in mathematics teaching is a creative and knowledgeable teacher. The teacher should personify a view of mathematics as a social subject, as a cultural product and as a language with both formal and informal ways of expressing itself. (Ernest, 1991, 1996) Moreover, the teacher's knowledge about different methods of teaching and insights in how the often very stable and limited conceptions of mathematical concepts are constructed within the learner, form an important foundation for the active learning approach. (Thompson, 1989) It is the teacher's job to organise the teaching situations and the curriculum in such a way that the learners are able to communicate, evaluate, develop and expand their mathematical strategies and conceptual network.

The role of the mathematics teacher is seen as that of a manager of the learning environment and learning resources with guidance and support from a non-threatening, communicative and inclusive environment. According to Rudduck (1996) the learner's views of the interaction with the teacher are strongly connected to the learner's view of how she is getting on in the subject. Moreover, recent research results support the hypothesis that there is a positive correlation between the learner's image of personal mathematical ability and the mathematical achievement of the learner. (Linnanmäki, 2001; Ruthven, 1987) The teacher's role in developing the learning culture is thus essential. (Seeger et al, 1998) A productive learning culture in the mathematics classroom is not necessarily established with imposed order and lots of teacher talk. If the teacher uses an imposed order as a means of helping learners learn, they may see this as an unreasonably strict code that distances the teacher from the relationships necessary for productive communication.

3. Open-ended and mathematically rich learning tasks are considered a prerequisite for the possibility of constructing a conceptually rich cognitive structure and a relational understanding of mathematical concepts.

This statement is strongly supported by a large body of research that shows the advantages of open approaches to mathematics teaching. (i.e. Baird & Northfield, 1992; Black & Atkin, 1996; Dance, 1997) However, the statement does not exclude the

need for procedural training and repetitive drill. But the traditional types of mathematical activity should be integrated in and subordinated to mathematical tasks embedded in meaningful and social situations which motivate the learner to be more active.

As a basis to the statement, let me introduce an example from the Norwegian KIM-project. In the KIM⁶-project the researchers noticed an alarmingly low level of competence in solving arithmetical problems, for instance $25 - 2 * a = 17$. This is probably due to a weak understanding of the arithmetical conventions. Traditionally, the cure to problems within this type of mathematical problem solving is considered to be more repetitive drill with the same types of problem. But research also shows us that this is not enough. The learner often makes the same mistakes over and over again. The KIM researchers decided to introduce a critical and social framework into the teaching process. Instead of doing lots of similar exercises by their own, the learners were urged to examine the correctness of mathematical statements, for instance the statement $7 + 2 * 3 = 6 * 2 + 1$. Then, the learners were invited to find appropriate ways to make changes in the discovered false statements in order to make the statements true and, also, to work and discuss the solutions with a partner.

The introduction of more open-ended learning tasks into the teaching of mathematics is not a straightforward process. Mathematical tasks that are open-ended, and thus more suitable for active learning, have a more unclear solution process and outcome than traditional tasks. The setting becomes more complex creating a situation that is not comfortable for a teacher trained to teach mathematics in the traditional manner. Moreover, the openness of the process may leave those learners feeling uncomfortable who are used to more closed and passive learning situations. Some will be overwhelmed with the details that the openness of the process can generate. The introduction of new types of tasks and methods of teaching should therefore be done in a carefully planned manner. Also, according to Ruthven (1987), changing the context of a task can have marked effects on how its difficulty is perceived by the learner.

4. The set-up of the classroom should support interaction in small groups, whole-class discussion and individual seatwork in accordance with the needs of the learner and the learning task.

The set-up of the classroom should be flexible to permit work and interaction in small groups. Learners should have freedom of movement as well as freedom of speech in the classroom. The culture of the discussion should be open to non-traditional patterns of interaction in contrast to the traditional pattern of discussions initiated by the teacher, and conventional forms of discourse. (i.e. Robertson et al, 1994; Seeger et al, 1998) The conventional form of discourse is often characterised by

⁶ KIM is an acronym for "Quality in the Teaching of Mathematics". Results from the research project are presented in Brekke, G, Grønmo, L. S. & Rosén, B. (2000). Charting mathematical understanding. Guidance to algebra. (written in Norwegian) Oslo: Nasjonalt læremiddelsenter (NLS).

questions asked for the purpose of evaluating learners' answers for congruence with the answers already known to the teacher. (Bartolini Bussi, 1998)

The more complicated the mathematical concepts and procedures become, the more frequent experience in using discussions is needed. Interaction creates connections between procedures and establishes links between procedures and conceptual knowledge. This process is enhanced if the learners are accustomed to thinking aloud during the discussions and to using monitoring questions like: "Can we describe this more precisely?", "What exactly are we doing now?", "What does this outcome mean, does it make sense?", "Can we express this differently?" and "Can we explain why we did this?"

The theme "Active learning of mathematics" in workshop-format: an example

Finally, to enter more deeply into the character of "active learning in mathematics" I shall outline the structure of a workshop that was included in the ICEL2000 conference. In this workshop my overall goal was to guide the participants in constructing a personal vision of what active learning in mathematics is all about. Further, I wanted to create a feeling in the participants of being personally involved in a process guided by the idea of the learner as an active participant in the learning process. I assumed that the workshop would involve a mathematically heterogeneous group of people. Hence, I used mathematical problems that, hopefully, would not be terrifying to anyone, regardless of previous experiences of mathematics. Unfortunately the workshop attracted just a few people and, hence, some of the planned activities were not acted out. Nevertheless, the discussions revealed a big interest in finding ways to change mathematics teaching into a more learner active mode.

The structure of the workshop was divided into three groups of activities that are described below:

Activity 1:

The main aim of the first activity was to establish a sense of community (Dance, 1997) in the group. Secondly, it acted as a trigger to discussion about the features of active learning of mathematics and, thirdly, it also illustrated aspects of multiple forms of representations.

All group-members write their names in big letters on pieces of paper. Two different colours can be used to expand the mathematical potential of the activity, for instance to highlight the mathematical relation between men and women in the group. The pieces of paper are arranged on the board or on a place where everyone can see them properly. The structure of the arrangement may be altered according to suggestions from the group. A group discussion follows about which mathematical ideas and representations were possibly developed by this activity. How can the activity be extended? What components of active learning of mathematics are found in the activity?

Activity 2:

The aim of the second activity was to illustrate how previous experiences, including conceptions of mathematics, attitudes and feelings towards mathematics are part of the learner's mathematical history and thus affect the way the learner conveys mathematics.

In this activity the co-operative method *Think-Share-Four* (i.e. Kagan, 1994) is used. First, *think individually*: select two or three favourite numbers, think of how these numbers relate to your life. Then, *share in a group of four*. Tell each other about your numbers. *These two steps are followed by an extended discussion in the group of four.* Compare and contrast the eight to twelve group numbers from two perspectives. Firstly, from an emotional perspective: Select *A Special Group Number*, one number that has some common connection(s) to everyone in the group. Secondly, from a mathematical perspective: Investigate the group numbers by using the procedural and conceptual language of mathematics, with symbols, pictures, graphs etc. and put the findings on display.

Activity 3:

The remaining time of the workshop was devoted to discussions about research findings as the basis for changes in the process of teaching and learning of mathematics. The following three statements were illustrated with examples and discussed in relation to active learning of mathematics:

Simultaneous change is needed in

- 1) the nature of the learning task (from closed to interactive and open mathematical learning tasks),
- 2) the structure of the classroom (from teacher-centred to learner-centred including proper arrangements of the learning-space)
- 3) the nature of the interaction (from an unilateral or a bilateral type of interaction directed by a teacher to a multilateral interaction among all participants in a group, based on the requirements of the learning tasks).

Some concluding remarks

The active learning approach looks at the process of mathematics learning in ways that can entail everything from small but conscious changes in teaching methods or pedagogical techniques to a total change in the teachers' personal philosophy of mathematics education. Mathematics learning as a process where the learner actively constructs knowledge acts as a fundamental guiding principle in these change processes. However, mathematics teachers who are accustomed to work traditionally have not needed much knowledge of how children learn mathematics. The focus of the teaching has been more on the mathematical performance than on competence and mathematical understanding. The teachers have been required only to explain to learners set sequences of procedures prescribed by textbooks. Thus teachers accustomed to teaching the traditional curriculum may lack knowledge about

mathematics learning and teaching methods that is essential to implementing fruitful changes in the classroom learning culture. (Battista, 1994)

From my own experience in teaching mathematics at the junior secondary level in Finland, and also as an educator of primary school teachers and action-researcher, I can tell that the road of instructional change is bumpy and that a teacher needs a lot of integrity to walk it. But in fact, looking back at the effects makes you think that it was worth it. As a teacher I think you should keep in mind that just as a single bad experience in class can produce a bad image of mathematics, a single good experience can provide the foundation for the development of a positive one!

According to Ernest (1996, 813) mathematics is an aspect of human activity and culture, which can bring pleasure, enjoyment and fascination. He stresses that all persons should have the opportunity to have such positive experiences. My personal opinion is that mathematics teachers could make this happen by establishing an active learning culture in their classrooms.

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