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School mathematical practices as experiences of identity work in relation to problem solving - A critical examination

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Abstract: *The key ideas in this paper have emerged out of sensitive interpretations of experiential accounts given by students and educators in school mathematics. The educators, inspired by social constructivism, expressed a special interest in developing students' responsibilities for learning and mathematical thinking skills as well as their abilities to solve mathematical problems. The paper illustrates that students' emerging identities as 'problem-solving mathematicians' connect to differences in their school mathematical practices during the school years.*

Keywords: case study, identity work, school mathematical practices, problem solving

Introduction

The key ideas in this paper have emerged out of theoretically informed interpretations of experiential accounts given by three students during lower secondary school and as adults, as well as by a group of educators. Within practices of school based teacher-research the educators articulated a special interest in developing the assessments of students' learning of mathematics and learning skills in mathematics, including mathematical problem solving.

The main results, which are presented below in three student stories, have emerged out of investigations into *how the students related to their school mathematical experiences, and especially to the problem solving activities.* The situated nature of the stories is illuminated with a portrayal that positions the teacher-research theoretically and institutionally.

The paper meets requests from both within and outside the mathematics education research community for sensitive research studies where students are listened to in order to understand and learn from their experiences and expectations (Burton, 1994; Corbett & Wilson, 1995). It complements other studies which discuss how students, described as 'mathematically successful' by the assessment system, may become totally disaffected, 'underachievers' and resisters of mathematics, as a result of their participative experiences within a usual school mathematics practice (see e.g. Boaler, 1997b; Nardi, 2003; Reay, & William, 1999). Furthermore, it meets the scepticism expressed by some (e.g. Wagner, 1997; Pring, 2000) concerning the value and validity of such educational research which does not involve the 'practitioners' themselves in a profound way.

Some conceptual clarifications

'Practice' is used to stand for the repertoire of actions and activities which constitute a community including the experiences and expectations of its members. 'Participation' denotes learning in a social sense as emerging within school mathematical activities both in and out of school. Being members in a school mathematical practice is defined as a reflexive process that both produces and transforms the participants' identities. Identity is according to Yvette Solomon (2007) central to any social theory of learning.

> As far as mathematics is concerned, [identity] is essential to students' beliefs about themselves as learners and as potential mathematicians and it has vital gender, race and class components (Solomon, 2007).

Furthermore, to be a member in a practice does not necessarily suggest that a person identifies him/herself as a contributor to the forms or contents of actions and activities. From the perspective of the individual 'practice' is neither a constant entity nor the same for all. A member might be observed (be positioned by an outsider) as passive, solitary or cautious but nevertheless be highly engaged (position himself) in the actions and activities which constitute a community, while another member in a practice may be very active and doing much but not be experiencing his membership as a strong engagement. A questioning person might for instance experience moments of exclusion in situations where his or her ideas are "not heard". Practice is experienced "as practice", i.e. as customary ways of behaving, but the related meanings may not be shared. So, as Boylan (2004) was able to illustrate, the same students that together with their teacher constitute 'a school mathematical practice' can be conceptualized as potential members in different 'practices'. Further, to open up for the possibilities of belonging as well as becoming; that is, to be members in and moving in and out of a variety of 'practices', the notion of 'identity' is interpreted as neither homogeneous nor static. According to Wenger's (1998) social theory of learning *'identity' is a constant becoming, a work that is always going on* as a layering of events of participation and reification by which experience and its social interpretation inform each other. 'Identity' is constituted by what a person *is* and by what a person *is not* in terms of competences manifested in sharing a common enterprise, values, assumptions, purposes and rules of engagement and communication.

> /.../ As we encounter our effects on the world and develop our relations with others, these layers build upon each other to produce our identity as a very complex interweaving of participative experience and reificative projections. Bringing the two together through the negotiation of meaning, we construct who we are. In the same way that meaning exists in its negotiation, identity exists – not as an object in and of itself – but in the constant work of negotiating the self (Wenger, 1998, 151)

Broker and researcher

The paper connects to two types of mathematics education research work and to the author's position within this complexity: to *school based teacher-research which aimed at changing local mathematics teaching and learning practices*, and to *individual, scholastic work in the form of a long-term interpretive story-telling and picture-drawing case study* (Bassey, 1999). For the sake of clarity I will hereafter use the acronyms LOR (local reform) and DAR (doctoral research) when referring to respective research interest. 'Student' is used to account for student-learners being both present and former learners of school mathematics during the course of the case study research. However, all members in LOR, the author as well as other educators involved, are conceptualized as learners. A significant difference between the two research perspectives is that students are positioned as legitimate participants in the DAR research process as well as its beneficiaries; in LOR more as beneficiaries. In DAR the approach is to understand the students on their on terms (Corbett & Wilson, 1995).

The research approach within DAR does not fall within a *scientific research paradigm* in the sense that the results reported here have emerged out of *a priori* decisions about what to systematically look for from a neutral observer's position (Ernest, 1998). The paper portrays *an interpretive learning process.* The LOR-school culture was familiar to me from my own earlier teaching there and from professional and private contacts with the educators. Furthermore, I worked closely with the LOR- educators and cannot consider myself to be an objective instrument detached from the school mathematics practices I was penetrating through the voices of the students and the teachers. On the one hand I acted as an *insider and research companion in the LOR-process*, for instance I wrote minutes of all 19 formal LOR-meetings. I also acted as a *broker* (Wenger, 1998, 104) and empathetic listener who introduced interpretations of student school mathematical practices as expressed in student interviews into the LOR reform practice. But my engagement relates as well to an *outsider research interest* as the whole LOR-process, and its material products, is used as empirical data for the doctoral research to which this paper connects.

The empirical data and the analysis

I conducted series of individual interviews with altogether 13 LOR-students during lower secondary school: in September and December year 7, in December year 8 and in December and May year 9. The students were asked to describe and comment on their school mathematical experiences and expectations, for instance in relation to recent ‘ordinary’ lessons. I also probed the students about their relationships to problem solving activities and to other activities discussed in LOR. As a result of methodological considerations within DAR three students, *Joakim, Kristina and Nette* (pseudonyms), from one classroom (out of three) were selected as key student participants in the case study research and were as adults invited to back-ward looking conversations. The adult interview focused not only on memories of problem solving activities at lower secondary school but also on mathematical practices more widely. My first round of interpretations of the lower secondary interviews was negotiated as well.

Primary empirical data analysed *for this paper* are 18 audio-taped discussions with the three key students and the full transcripts. Secondary empirical data, used for situating the student accounts theoretically and institutionally within the reform practices (see next section), are full transcripts of interviews with six LOR-teachers, minutes from LOR-meetings, lesson observations and other various LOR-documents, as copies of project work reports.

Wellington (2000) recommends a researcher to select various key-informants if interviewing is used for studying different perspectives on educational issues. In the case study considered here it was not, however, an issue for the research approach whether the key student participants represent ‘a cross-section of students’ or not. There was no *a priori* research intention to categorize or label the students or to look for connecting explanatory patterns in their stories, but to understand the school mathematical experiences they describe.

Personal accounts are in the analysis considered “as practice” (Säljö, 1997) and are interpreted neither as separated from the actions and activities that constitute the communities where they are situated nor as a window into the minds of the persons. The accounts are considered as an artifact that also holds important information about the positioning of students (and educators) as ‘identity-workers’ together with the practices which informed the production of the accounts. For instance, I take Joakim’s ways of using “*we*” in expressions like “*we* go through”, “*we take problems* on the black board”, “*we understand* how to calculate” as indications of a membership in a community where his mathematical knowing was acknowledged to become reified as the socially accepted knowing of a “mathematically able person”. Joakm’s knowing is most probably different from the knowing reified in the case of Kristina, who in expressions like “*they* reshape the rule into a form that *you are* *expected to understand* when *they write* it down on the board, and *we copy*” seems to position herself in a marginal position to this community.

Theoretical and institutional positioning of the Local Reform

LOR started when a group of mathematics teachers met an ‘identity crisis’ in their teaching practices and looked to research/researchers for collaboration around issues related to potential strategies for change. The collaboration in LOR followed an action research design with learning cycles of individual exploration, joint reflection, evaluation and decision making. LOR continued for a three year period which coincides with the school years and the roughly 340 lessons in mathematics the students included in DAR experienced at the lower secondary level.

Epistemologically LOR was strongly inspired by a version of constructivism where the social is seen as influencing the cognitive structuring and, hence, the knowledge development of the individual (Björkqvist, 1993; Häggblom, 1994). LOR thus connects to cognitivism and to the constructivist theories of learning and knowledge that in the 1980ies and early 1990ies had gained a strong foothold within the mathematics education and research discourse both in Finland (e.g. Malinen & Kupari, 2003) and elsewhere (e.g. Davis, Maher & Noddings, 1990; Steffe & Thompson, 2000). Constructivism was then commonly referred to as a socially accepted ‘normative good’ and was therefore a source of power in movements to transform learning. The normative interpretations of constructivism within the community of researchers appear for example in such ‘learn-the-thinking/assess-the-thinking’- recommendations as the following:

> /…/ thinking skills must be the focus of instruction in mathematics (…) assessment procedures need to be developed that portray not only the number of correct answers students can produce, but the thinking that produced those answers (Romberg, 1993, 107).

In LOR *a constructivist perspective* on learning was considered in parallel with *a problem solving perspective* on mathematics teaching and assessment practices. This approach was supported as well by the 1994 national curriculum which argued that students come to know mathematics through connecting new to old knowledge, by restructuring models of thinking and acting and by training rational and exact thinking. The curriculum further declared problem solving and the internal logic of mathematics as “the most important principles for mathematics teaching" (NBE, 1994). The LOR teachers were also aware of students' low level of problem-solving performances in nation-wide tests (Björkqvist, 1995) and of international research on problem solving. For example, in a lecture entitled "New perspectives in assessment" Jeremy Kilpatrick (1993) indicated *the thinking process to a problem solution*, including both explaining and defending the problem solutions, as an important ingredient in assessment practices. Thus, behind the argument is a tacit assumption that an emerging awareness of 'efficient' thinking processes via assessment would advance the discovery of mathematical structures and be further advanced through problem solving as an assessment ingredient.

Teaching was within LOR conceptualised as identifying and conveying mathematically valuable 'thinking activities' to the students as well as *guiding students to discover by themselves* the significant mathematical knowledge structures situated within these 'thinking activities'. The process of discovery was expected to resist the meaninglessness connected to learning mathematical concepts and formal structures in a ritual fashion. The LOR teachers described themselves as *structurers*, *guides*, *helpers* and *control* persons who monitor the thinking of individual students in order to prevent 'mislearning' to happen and to support knowledge transfer (Röj-Lindberg, 2006). For example, the teachers used cognitively/-biologically focused metaphors like "to fill gaps", "to give students mathematical eyes" and "to connect their brains, make them think independently". Learning was conceptualised as mediated in the course of social interaction, especially through 'thinking/reflection-questions'. Such questions were for instance 'How are we supposed to do here', 'How did you think to come up with that answer to the task', 'Can you explain the rule' and 'Is there another way to do/to explain'. This *learning-by-discover*y-*of-structur*es- approach was in the local school curriculum pointed out as especially important to consider by teachers in relation to students with "personal qualifications and a personal interest to study more theoretically advanced mathematics". Important for the theme of this paper is that the local curriculum, and most likely, even if perhaps implicitly, the LOR- teachers themselves, thus seemed to accept marginalisation of students positioned as disinterested or as lacking "personal qualifications" in "discovery-of-structures"-.situations when the formal and abstract nature of mathematics was to be the overt attention for teachers and students.

Problem solving activities as tools for discovery and assessment

The LOR- teachers expressed beliefs in problem solving as an art of individual discovery guided by teaching, and in that a student *in the course of problem solving activities* could be assisted to ‘see’ various mathematical structures implicit in a problem, to ‘see’ mathematical structures both as meaningful as such and as meaningful for use on a variety of problems, and to ‘see’ the need for taking personal responsibility for learning. Consequently, as one potential way out of their ‘identity crisis’ the teachers introduced some special types of problem solving activities both for formal assessment purposes and as cognitive and social ‘eye-openers’ for students. The teachers also expressed an intention to *coach the students in problem solving* with the help of problems that transcended the ordinary closed, ‘textbook-type’ problems. In short, problem solving practices was considered by the LOR- teachers both as an end to be achieved *from participation* and as resources *for participation and later use*. ‘Usual school mathematics’, where solving textbook tasks of the predominantly closed character is the core pedagogical tool, continued however over the school years. This is visible in both the teachers’ and the students’ accounts. The students refer to qualities indicating the closed condition of the activities they regularly took part in, for example in utterances such as “to be sure [teaching in years 7,8 and 9] followed the textbook”, “importance of a right answer”, “rules you must follow”, “lack of space for you own opinions” and “already decided how the solution to a problem must be, how to calculate, you have to try to manage [the problem solutions] by yourself”.

LOR introduced two main types of tasks which combined problem solving with formal assessment: firstly, *mini-problems* used on average 3-4 times a month during school years 8 and 9. These problems, 42 in total, were individually solved during lessons in 10-15 minutes, occasionally as group work. In school year 9 the mini-problems were selected especially with those students in mind who were prospective participators in the upper secondary advanced mathematics. Hence, students like Kristina and Nette, who in year 9 chose the basic mathematics grouping[1], met fewer mini-problems than students like Joakim, who chose the advanced mathematics grouping. In May year 9 Joakim refers to the “many tests and mini-problems during the last two months”. The following task is a mini-problem used in year 9.

[1] Here the reference is to the separation of the mathematics curriculum in Finnish upper secondary schools into a basic curriculum and an advanced curriculum. In LOR groups mathematics teaching was differentiated and new student groups were formed already in the midst of grade 9. The new groups were to be formed according to future study plans of the students, i.e. vocational mathematics, upper secondary basic mathematics, and upper secondary advanced mathematics and not primarily to be an “ability-based’ grouping. In reality, however, a student who chose upper secondary mathematics track was positioned as ‘more mathematically able’ than others.

> *HOW MANY NUMBERS?*
> How many numbers between 1 and 5000 end with the digits 73?
> How many numbers between 1 and 23.000.000 end with the digits 73?
> Formulate a general rule that tells how you can determine the amount of numbers!

Secondly, *projects* offered to students once or twice each semester, 9 projects in total, including one thematically “Open project” and one “Group project”. The projects were homework tasks which extended over one to two weeks in the form of mathematical investigations with reference to pure mathematics, to a semi-reality or to real-life. Kristina’s description of project work as a “mixture between to calculate and problem solving” is illuminating. So is her teacher’s reference to projects as either for “finding connections and patterns” or for “working with something where connections and patterns are not that salient”. In some projects formal mathematics was applied in coming to grips with empirical facts about hobbies or societal issues like unemployment, politics and economics. Other projects were of a pure algebraic (e.g. “Number chains”) or geometric (e.g. “Regular polygons”) character. Grades were awarded for reports on the basis of generalised level descriptions. Among the criteria used were “discovery of connections and patterns”, “correctness and perfection”, “creative development of the problem”, “done with interest and care”, “high amount of time on task” and “length of and clarity in the report”. In relation to the group project and the open project students were as well expected to hand over initial plans “not exceeding an A4-page” considering “solution plans” (Group project) as well as “how ‘mathematical’ are the tasks; how will mathematics be used” (Open project). According to Nette students generally “did not account for it”, i.e. the final reports were silently handed over to the teacher without presentations or argumentations in the classroom. By and large students’ as well as teachers’ accounts illuminate a general expectation that students work with projects and mini-problems not because of curiosity or socially approved need for the problem solutions but in order to display proficiency in the service of individual assessment in the same vein as conventional achievement tests. However, and contrary to conventional tests, the awarded project grade related to both inner-mathematical (e.g. connections and patterns, correctness) and extra-mathematical aspects (e.g. time, interest, care, length).

Joakim, Kristina and Nette: potential problem solving mathematicians?

In the beginning of year 7 I positioned Joakim, Kristina and Nette as more mathematically successful and more engaged than the average student. Thus a suitable general description of their problem solving identities at this point in time could perhaps be *promising novices* or *potential problem solving mathematicians*. Their primary-level study marks labelled them ‘excellent’ (Joakim) and ‘good’ (Kristina and Nette). Their achievements in an introductory mathematics test were described as ‘high’ and ‘close to high’, and with 98% (Joakim), 77% (Kristina) and 84% (Nette) correct solutions respectively. These results can be compared to the four group-based averages within the age group: 56%, 62%, 81% and 85%. The last (85%) includes the results of Joakim, Kristina and Nette. Right through lower secondary school their awarded grades in different types of assessments, including the problem solving activities mentioned above, generally continued to be on a high level, very seldom below 8 on a scale from 4 to 10. So, generally over the lower secondary school years they solved school mathematical problems in a manner their teacher expected them to do. Up until the midst of year 9 they were classmates in mathematics in what their teacher described as “a group of high-achievers”. As adult Joakim conjectures that “more than half of the group were above 8 in average”. All three continued after year 9 to the same upper secondary school; Joakim to advanced mathematics courses, Kristina and Nette to basic mathematics courses.

The first interview

In the first interview all three expressed a strong commitment to learning mathematics and to developing the knowing, socially as well as cognitively, that they thought was expected from them and would be useful for them both in and out of school. They seemed to agree on that mathematics in school is an important and necessary subject, with some positive aesthetic features, and, contrary to findings in other studies, that its contents need neither be boring to work with nor impossible to understand. School mathematical knowing was further expected to, at least partly, afford functional knowledge for use in their everyday lives and futures. All three sensed future success in mathematical practices as within their reach. Nette refers explicitly to a future as veterinarian. Joakim and Kristina were generally confident in the significance of learning mathematics. Also, they all expressed a general reliance in the teacher as mediator in their cognitive activity of individually ‘discovering’ intended, and directly useful, mathematical meanings. During lessons commitments to procedural and conformist actions like “sit and attend” during the “go through” teaching procedure were expected to result in understanding “how to calculate the stuff the teacher has gone through”.

Self-concepts in relation to school mathematics

A person's self-concept can be described as a question of empowerment and sense of agency, and originating in a sense of being able to initiate and carry out actions on one's own or by one's own effort (Bruner, 1996/2002, 54). The three students' self-concepts in relation to school mathematics can thus be interpreted as informing as well about their experiences and expectations related to possibilities to work on developing an identity as a 'mathematicians' and 'problem solvers'. Results from a self-concept survey carried out in August year 7 indicated a remarkably stronger self-concept in relation to school mathematics especially for Joakim, but also for Nette, than for Kristina. A renewed self-concept survey in May school year 9 showed a slight movement to a clearer sense of negativity related to mathematics in the case of Kristina. In the case of Nette, however, a much more remarkable negative change in her relationship to mathematics emerged during lower secondary school. Her share of positive or neutral answers dropped from 82% to 35% (n=17). There are no renewed self-concept survey data on Joakim.

Based on multiple readings of the six interviews per student taken as a whole, the following brief stories shed further light on the emerging differences in the students' school mathematical practices. According to the high marks awarded in formal assessments, including problem solving activities like mini-problems and project works, it can be argued that the three students during lower secondary school were conceptualized as more 'mathematically successful' than average students. The remarkable different trajectories of participation illustrated by the stories below indicate however that there was not a parallel success in their personal identifications with the school mathematical enterprise as a whole.

Joakim's story: Mathematics was never a problematic subject

Joakim's interview accounts indicate that he during lower secondary school did not always enter the mathematics class "with a smile on his lips". However, he seems to have continued to strategically consider school mathematical work as much from the perspective of usefulness and importance as from the perspective of personal enjoyment.

Both Joakim's own accounts and assessment remarks by his teacher indicate a continuously successful commitment related to learning mathematics in school. His school mathematical trajectory ended up in an excellent matriculation examination which granted him open access to a Faculty of Technology. Joakim's was, as it seems, always engaged in the very real issues that concerned him, also when solving closed school mathematical problems of textbook type. He persistently expresses a sense of being in control over his identity work as learner. Also, he relates positively and in a strategically accepting and loyal manner to the teacher/teaching practice as well as to performance oriented assessments. The following statements are very indicative of his relaxed, “sit/take-it-easy/attend” relationship to mathematics in school. He has been successful and has no reason to believe that his learning policy will work against him in the future.

> (...) When you have had mathematics for such a long time already, I have had fairly high grades for a long time, then it is perhaps natural that you think that it will be all right (sw: nog ordnar det upp sig). (…) To ask the teacher how he will assess us is important so you know what is significant to concentrate on. (Joakim, December, year 8)

From Joakim’s perspective the teacher addressed the students in an including and sensitive manner which worked to guarantee his position in the community. Thus, he participated in activities where such rules he considered to be of a wider legitimacy were negotiated and where his contributions to the construction of the significant rules and their meanings were adopted. He positions himself as a ’rule-producer’ as well as a ’rule-adopter’ and seems to accept and meet the terms of, but not be confused or restrained by, a structured and rule-bound nature of the teaching of mathematical procedures and a need of memorising rules for later use in problem solving and assessment. He is definitely not disturbed by a ‘rule-following’, sometimes monotonous and unchallenging, nature of school mathematics. Joakims utterances strongly indicate that he positions himself as member in a community of negotiators where the teacher was included and also his expectation of the community as a space where his knowing is seen. He locates himself among those “*we*” who ‘see’ and are seen to ‘see’, who cope with the teacher/teaching and who know how “smart” persons legitimately act in the mathematics classroom. Furthermore, he talks about his willingness to abandon a successful problem solving strategy in favour of “the one the teacher shows” if the teacher’s strategy takes him more effectively or rapidly to the correct solution. It is thus sensible that he some weeks before leaving comprehensive school finds no strong reasons to doubt that the rules and solutions he is looking for when solving mathematical problems are there “somewhere in [his] head”.

> If I know that I should have the solution somewhere in my head I sit and try my way forward, but if I notice that, no, I cannot do this I give up quite easily. But I usually think about what rules there are for this type of tasks and then I try to apply them and then I see if it is possible with such an answer (…) If the tasks are difficult there are always others who cannot do them and we usually take them on the black board (Joakim, May year 9))

Joakim knows about the conceptual resources in mathematics. He knows there are rules to look for. When he finds them he knows to apply them. A bigger, but not impossible, problem is to discover order in a mathematical text, to decide "what type of task" it is. His tolerance for failure is sometimes minimal. But he also expresses confidence in never to be the only one who doesn't 'see' a problem solution and in always to understand the explanations of the teacher. Thus, he sees no need to worry even though he is aware of that he might meet a dead end in his mathematical thinking. "To give up" indicates that the tasks are *in essence* difficult, not that he and "others who cannot do them" are unable to "think" or run the risk of exclusion from the community where the legitimate rules of the mathematical game are constructed. As an adult Joakim looks back on him as one in a group of "tough guys" who coped with the advanced mathematics course and he refers to the "many wise ones in the group who performed well when they wanted". To <u>not</u> 'see' or succeed in mathematical problem solving was simply never an issue for him.

Whatever was actualized by his mathematics teacher during lower secondary school Joakim always wanted to know it as well as possible and without further questioning of the type 'Why learn this?'. He did not, as he said, "in [his] spinal marrow have any immediate need of questioning do you really need to know this". His school mathematical identity work seldom seem to have originated in any need to think differently, "to cross the borders", and perhaps ask 'what if we do it another way?'. To him school mathematical teaching and learning was a closed and strongly, but not blindly, aligned process of "paper filling" and knowing "the whole paper". In line with his accounts during lower secondary school he concludes that his goals for school mathematical knowing were ambitious, but not restricted to "survival", yet to secure knowing of the mathematical content of "the filled paper". Mini-problems and project-works were informative in the sense that they expanded his view on mathematics but were anyhow marginal to the contents of the usual school mathematical discourse.

> (…) if you see the content of the teaching of mathematics as an A4-paper you survive (klarar sig) on knowing half of that paper. I wanted to know the whole of that paper. But I never gave so much thought to considering what if you would have crossed those borders a little, even though mini-problems and project-work did show a little of what mathematics can be (Joakim, adult interview).

The 'face of mathematics' in school was seldom of the social and human nature he as an adult experiences in his profession. Now, as an engineer, he expects mathematics to be "something like that (sw: lite ditåt)", and a tool "where the social and your personal decisions matter”. In school it was more like “playing around with the models and rules you have learnt”. But it was also to excitingly imagine a real need, a pseudo-reality, for the mathematical models and rules and to participate in a stimulating game of success where he was rewarded, in control, and never felt any real risk of being excluded from the community of “smart” persons. His acceptance of mathematics in school was based on ‘seeing’ its learning content as secure, hierarchical and absolute, but also on ‘seeing’ its future meaningfulness in his life and on support at home, and more importantly, on confidence in the consistent and challenging support at school. The school support reminded him over and over again about his membership in the community of “we who know these things”. Memberships in school mathematical practices seem in the case of Joakim to have reconciled during the lower secondary school years into a nexus of identification as an “able” ‘problem-solving mathematician’.

> When you discovered a more difficult level, the level of great distinction (sw: laudatur nivån) the teacher always showed a [special sign] of support (…) then the LOR- project (…) the structure with a theory booklet and a training booklet, very good and clear (…) the teacher urged us to carry the theory booklet around, he said ‘if the training booklet is burnt you can always get a new one’ (…) and these mini-problems and project works (…) I felt privileged (…) we had something the other classes did not have (…) more difficult than according to the curriculum (…) it was something to be proud of, like the image of the class. (…) you became like, uups, we are in fact rather able, we who know these things. (Joakim, adult interview)

Such special signs, rituals, actions and activities as described above, indicated from Joakim’s perspective a process of inclusion and an extraordinary challenge, support and concern by his LOR- teacher for helping “able” students like himself solve mathematical problems. However, from another student’s perspective such signs might as well have reified as institutional barriers which restricted participation and identifications as ‘problem solving mathematicians’. As the stories of Kristina and Nette will illustrate, what is meant as affording inclusiveness might imply the very opposite. For instance, the [special sign], which from Joakim’s perspective acted as a sign of support for “discovering the level of great distinction”, mediated from both Kristina’s and Nette’s perspectives knowing connected more, or perhaps solely, to entertainment than to learning legitimate mathematics. It was “some kind of teacher’s joke” confirming social bonds within communities constitutive of Others than themselves.

Kristina’s story: I liked mathematics, but it was not for me

Neither a positive relationship to school in general nor mathematical performances awarded by ‘good’ grades was enough to positively influence Kristina’s sense of agency related to school mathematical practices during lower secondary school. Her utterances over the years support a description of her identity work as ‘fights of existence’ enacted on home grounds with family members as well as in the mathematics classroom: to maintain participatory positions not only connected to ‘who am I in relation to mathematics’ but also to ‘what is mathematics for me’.

Both at home and in school she seems continuously to have put her own mathematical activities alongside those of others and to have tacitly asked herself ‘do I belong to the community of mathematically able persons or not’. For example, she refers to her mother as a mathematical outsider. This statement from year 7 is indicative “I think I understand [mathematics] better than my mother”. So, in relation to her mother she positions herself as an ‘able’ person mathematically. But in relation to essential mathematical negotiations during lessons she locates herself in a marginal position. She doesn’t position herself as a contributor and constructor of the mathematical learning content in situations where “finding out the rules” is the focus for overt attention. The utterances below indicate negativity in her engagement as she describes herself as a person in the margin of such an ‘discovery-of-mathematical-structures’- activity which was considered as a significant learning activity by the LOR-teachers.

> /…/ you at first try to find out the rules by yourself, or like the group. <u>I</u> usually don’t find out, in my opinion it is mostly the boys (…) then [the teacher] says whether it is right or like wrong (…) then they kind of reshape the rule into a form that you are expected to understand when they write it down on the board, and we copy. (Kristina, December, year 7)
> /…/ [the teacher] told us in the beginning of lower secondary about some [LOR-project] and that it is the right and proper thing that [teachers] should get the pupils to think independently as much as possible and that is why [the teacher] asks us like how to count things and not tells us how to count; he kind of lets us discover an answer by ourselves, I don’t maybe discover any answer so often but others do it. (Kristina, December, year 8)

She refers to the act of rule discovery as a solitary act, but socially situated within ‘a “they”-group’ wherein answers are discovered and rules are reshaped. She does not consider herself as a knowledge maker. She copies the rules from the board, and the teacher monitors the legitimacy of everybody’s work. Copying from the board is a legitimate action. On the other hand her accounts indicate acceptance of a position among ‘the rule-adopters’ and among those “we” who copy the rules in order to <u>later</u> ‘see’ their mathematical and social meaningfulness. However, in the midst of year 9, and forced by the instructional system to actively position herself as either “a long or a short mathematician”, this ‘acceptance of delayed meaningfulness’ of hers may have turned into surrender to the fact that impossibility of secure understanding was an ingrained aspect of her identity. At that time she realized that she is not and never will be capable of the rapid construction of new meanings required by the constant flow of “new things” during lessons, especially within a group for learning “advanced mathematics”.

> /.../ I did not manage to keep up during the lessons and then you lag behind and cannot concentrate fully on the new things (…) you kind of need to understand that one thing before you move to the next thing so your brain can manage to keep up (…) you need just one disappointment and then you think, okay, this is not my thing (…) you had to make a choice, are you a long or a short mathematician (…) I realized that I am a short mathematician, that advanced mathematics is too difficult for me. (Kristina, adult interview)

During the school years and as an adult Kristina never explicitly opposes her teachers and their teaching practices. On the contrary, she clearly expresses confidence in her teachers and she aligns strongly with the closed condition of mathematical practice (Röj-Lindberg, forthcoming). The closed condition, which includes learning through the textbook and assessment via textbook type tasks, she argues as inevitable for coming to know the legitimate mathematics which is negotiated during ordinary lessons. Over the school years she identifies positively with mathematics as an important school subject and as a tool to solve a variety of problems, including project work, in an aesthetically, controlled and orderly fashion. The “boring” aspects she explains as related, among other things, to lack of time to develop the expected mathematical competence. Mathematics is “fun” in situations where she knows “all the tasks”. It comes as no surprise when she as adult refers to her liking of the orderliness of school mathematics, for instance of the “thick, white and orderly” textbook, and that she describes the rule-bound activity of applying equations as ”such a fun”. Furthermore, her accounts indicate willingness to accept also a ‘non-human’ face of school mathematics characterized by a pressure to perform and by absoluteness: she describes mathematical activity as correct/incorrect resulting from right or wrong thinking and never ‘something in between’; and also by a repeatedly constraining rush that contributed to her marginal position during lessons. The tempo with something new “gone through on the board” almost every lesson demanded a constant alertness of her in order to understand the “gone-through”- mathematics, which from her point of view was mathematics discovered and structured by others. A core relationship to school mathematical practice as one of ‘responsible acceptance’ and ‘loyal engagement’ can be interpreted from statements given in a negative voice such as the following in December year 8 “I would like to avoid mathematics in school all together because it often is such a boring activity”; combined with statements indicating her strong sense of responsibility. For example in December year 9 she emphasizes that when taking part in school mathematical activities ”you cannot just think of those things that are amusing, mathematics is important you know”. Her loyal engagement is echoed as well in the adult interview where she remembers mathematics as a school subject she liked very much

> /.../ when you succeed and can keep up with the new things, mathematics is one of the most enjoyable subjects there are in school. (Kristina, adult interview)

In sum, in Kristina’s school mathematical practices ‘joyfulness’ came out as secondary while carefully “following the teacher” was of primary importance together with hard and solitary work. This was an approach she expected to grant her possibilities to come to know the mathematical concepts and formal structures the teacher wanted students to understand. However, her accounts also show a growing ambivalence whether mathematical knowing really will be important and useful for <u>her</u> future. Therefore I was not surprised that her school mathematical trajectory finally was an outward one. Despite a fairly successful matriculation examination it came to a point when she discarded her plan to become a biologist because of the closed condition of biology similar to that of mathematics. Looking back on her school trajectory as an adult she argues that Pedagogy became her final choice for further studies precisely because the practices of both mathematics and biology pronounce “here are <u>the</u> right answers”. Her compliance became more and more reluctant until she finally turned her back to the practices of mathematics and biology because these subjects afforded its learners a narrow frame for creativity and no social spaciousness. The legitimate face of mathematics emerges within “given rules, types of problems and solution models”; it is not a subject “where you can discuss your way through things”.

> /.../in mathematics the frame for creativity is rather narrow because there are the given rules and the given types of problems and the given solution models (…) you cannot in school be expected to creatively find the models, someone must give them to you (…) but of course you need a sort of creativity to find the solution model that fits to a problem (…) most important that the different models for solving different types of problems are presented to you (…) that is the basis, that is mathematics, that is rules and you have to learn the rules (…) the dilemma is with the quick tempo that, uups, wait a minute, what problems have we gone through so far (…) they just increase and increase and increase and it becomes chaos and too much (…) you realize that it is not a matter of course that I understand mathematics. (Kristina, adult interview)

In lower secondary interviews Kristina described project work and mini-problems as practices with lesser legitimacy and as fundamentally outside of usual school mathematical practices including mastering “rules and systems”, “what we are busy with in the theory booklet”, ”things to remember, different formulas and ways of calculating”. In these practices the security of mathematical knowing is granted cumulatively since “some little more can be added all the time, so then it is easier to understand those more difficult tasks”. To Kristina such practices seem to have reconciled during the school years into what can be described as ‘creative combinatory practices’ wherein mathematical structures, rules and models, must be learned for certainty in advance of, not through, problem solving. This problem solving competency includes as well coping with a constant risk of loosing ownership of meaning and being excluded by the instructional tempo together with a confusing and chaotic multitude of rules and models. The dilemma is, according to the adult Kristina, “that it is not possible to lock in creative thinking in boxes as you can do with rules and models”. A competent ‘problem solving mathematician’ is constituted by his or her capability to <u>internalize</u> rules and models offered by significant Others in ‘a “they”- group’, and to construct some sort of ‘typology for seeing’ out of these. That is, to understand how to differentiate between types of problems, to remember rules good for each type of problems and to know how to apply these rules for correctly solving the actual problem.

Nette’s story: Mathematics is for Others, I don’t have a head for it

The negative trend in Nette’s identity work which was visible in the self-concept surveys mentioned above was clearly evident as well in her utterances both during and after the school years. Her experiences were frustrated to such an extent that she, within the first minutes of the adult interview, wanted me to verify her hope that my aim of our meeting was <u>not</u> to make her “count mathematics”. Later she wraps up her negative relationship to school mathematics in a claim of her “lack of a head” for a difficult and uninteresting subject, much of the legitimate content of which anyhow has turned out to be useless and without importance in real life. She offers solving equations as an example of such “useless mathematics” and contrary to “everyday mathematics”. She further claims that she had to let go of an initial life plan to become veterinarian; undoubtedly including mathematics and a need of coming to know mathematics during the school years; mainly for these reasons. As adult she “goes to school again” and thus in some sense re-experiences her own frustrated identity work to be and become a ‘mathematician’; now getting her own frustrations confirmed through the eyes of students of hers. At the time of the adult interview Nette has a university degree in Developmental Psychology and is working at a lower secondary school with students in participatory difficulties related to mathematics. Nette's story is illustrative of how a strong trust in the usefulness of usual school mathematics and in always coming to know what is expected by attending and enquiring, is slowly demolished and turned into a low self-concept related to mathematics. It also especially illustrative of how societal biases about *a priori* cognitive constraints in coming to know mathematics, like 'to do mathematics you need a mathematical head', may be produced and reproduced in the course of social interaction to outlive reform efforts for expanding students' mathematical problem-solving competencies.

In September year 7 the interview indicates a strong commitment to take responsibility for learning mathematics by discovery in a solitary way. Nette's evident support of the aims of LOR is clear, and it continues to be so over the school years. In her words, she is content with a change from "just counting the pages they gave"- mode of participation at primary school into activities that anticipate students to "really think independently". She also puts confidence in school mathematics as a practice capable of affording her future successes in situations both in and out of school. Her way of using "we" indicate as well that she looks upon herself as both a 'rule-producer' and 'rule-adopter'.

> /.../mathematics is important, useful in all subjects (...) to be a veterinarian you need long mathematics courses in secondary school (…) all lessons look alike (…) we have theory when we write down the things we are doing (…) he asks different things (…) then we may count freely from the book (…) last time we did x times x and such. (Nette, September year 7)

Her strong alignment with a 'thinking/assess the thinking'- practice might however have worked as a roadblock for positioning herself as a 'mathematician' in the long run. In fact, her accounts in the first interview already illustrate this roadblock as an experienced participatory dilemma. From Nette's perspective it is definitively an unacceptable position if solving mathematical problems means to remember and to efficiently think about and apply rules and models, <u>without</u> a coherent image of the mathematical structures she is applying including arguments for their use. As will be further discussed later, Nette's accounts indicate, however, that the mathematical meanings of these structures appear as more and more strongly to be a matter of construction for Others, within practices where she is not a member, but for her to 'see'.

While Nette generally relates in a loyal and respectful manner to the teacher/teaching practices, she clearly indicates resentment of situations where she is left to struggle for a coherence related to the mathematical structures by herself. This might seem as a contradiction as ‘independent thinking’ is important for her. But, as it is the teacher who “has the rules”, it is a sensible wish of hers that it is the teacher who also should fulfil an associated and self-evident obligation to <u>transmit</u> their accurate meanings to students with the help of clear explanations. According to Nette’s accounts, such practices would grant students the secure knowing which they need to become able mathematical performers and problem solvers. “The teacher has to see to it that you (sw: man) really understand what he means (...) without you asking for it (…)”.

From Nette’s perspective the struggle for coherence is accompanied with feelings of insecurity due to the absolute nature of mathematical knowing. There is no ‘twilight space’ open in ‘legitimate mathematics’ for neither hypothetical thinking nor real collaborative work and investigations. Legitimate school mathematical work she describes as mostly nonsensical and tedious, but, on the other hand, it cannot *in essence* be of a really investigative, social and playful nature either. She concludes the non-human face of legitimate mathematics as inevitable. Mathematics is “numbers and rules (…) a lot of figures and signs to and fro” and it is always about “rightness or wrongness”. It is a socially important but dull subject “you cannot avoid” and “you just have to put up with” despite a more and more overwhelming disinterest.

Over the school years Nette’s accounts are strongly supportive of variations in the mathematical content and of the openness within problem solving activities like mini-problems and projects. Especially project work, even if purely mathematical like “Number chains”, afforded legitimate space for aesthetics and for creative knowing to be seen different from within usual school mathematics. She seems however, to an increasing extent, sense that formal grades awarded on the basis of the ‘mathematical’ aspects in her thinking is primary in legitimacy to formal grades based on such extra-mathematical aspects. On the other hand she opposes usual school mathematical activities that expect her thinking to concentrate on “just to count all the time”. And this is precisely the competence that school mathematical practice generally awards as legitimate and expects from her ‘thinking activities’. By and large her accounts indicate legitimate thinking within usual school mathematics as different from the meaningful and questioning type of thinking done within an investigative project work and when solving problems mathematically that really matters to her. But because such situations are rare and kept as separate, even if included within school mathematical practice, she identifies mathematics in school as not allowing her to become 'absorbed in doing' in ways she prefers in order to make sense and to be interested. When explaining in May year 9 why she finds mathematics lessons "very tedious" she indicates a total resignation and annoyance with such a disempowering practice. The accounts below are very illustrative of Nette's identity work and struggle during lower secondary school for being and becoming 'a questioning problem solving mathematician'.

> (…) simply not interested in this subject (…) cannot manage to see it as interesting (…) never liked mathematics, always found it boring (…) it has nothing to do with the teacher. It is just that I don't find it interesting at all. (…) if you are interested in what you do it is simple to do it (…) without interest in it, it feels only painful. It is like you lack the strength to do it (...) I don't have the strength to concentrate on what he goes through (…) rush all the time, no difference if you have understood what it is all about or not, like, you are just supposed to go further (…) I need time to understand all the different things (…) just sit there and what he says goes out of the other ear, it doesn't end up in anything (…) I don't succeed in tests because I have not had the strength to cope with what he has been teaching (…) mathematics is such that even though it is different things you go through it still is just to count all the time, it is no difference, it is so boring, it is like the same thing all the time. (…) the lesson would be unbearable if you wouldn't have someone to talk to. (Nette, May, year 9)

The persistence needed for doing usual school mathematics has simply vanished; Nette indicates small-talk with friends is her main empowering space for participation. As adult she still vividly remembers that such interactions were however considered as disturbing the legitimate participation, which was a matter of acceptance of the silent and solitary thinking for correctness in doing tasks "up and down" from "papers (…) super dull textbooks that all look alike".

> You cannot question lower secondary school mathematics (…) one plus one cannot be 'maybe something', it is two, fullstop. If you say one plus one is three it is wrong (…) mathematics is right or wrong, if you call this into question the teacher thinks that you try to escape (…) you ask why, then you get an explanation that "it is just so" (…) teachers are afraid of noise, or of one student counting while the other is copying (Nette, adult interview)

In the adult interview Nette jokingly tells about her total repression of memories from lower secondary school mathematics. Her jokes can however be interpreted as related to reified identities of neglect and diminishing trust in teacher/teaching practices and of being constrained in a struggle for both making her knowing visible and expanding her problem solving competences. This interpretation is based on the fact that she remembers a great deal of constraints related to access to participation, especially the following two incidents. Firstly, the sense of "stupidity" related to the regrouping of students into "those who knew and those who did not know mathematics" in the midst of year 9. Secondly, the frustrating incident in the spring term of year 9, i.e. after the regrouping, when she together with a friend grappled with sense-making 'why'- questions, but were met in a discouraging manner by their teacher. She quotes how the teacher commented on their request for more explanations and arguments for participation with "all others understand, how come that you don't understand". Her frustration over the incident indicates how a thoughtless comment from a teacher may be reified into disengagement and a silencing barrier for participation from a student's perspective. Also, as neither 'not understand' nor 'be irresponsible' were recognized conditions in Nette's identity work, or connected to any sense of 'lack of ability', the attitude of blaming the pedagogical approach that adjoins her description of silencing incidents during her school years, appear as sensible. Nette's blame of the usual school mathematical practice may be interpreted as her way of defending her sense of agency related to mathematics.

> Not understanding is a pedagogical problem (…) you cannot look understanding up in books or on the Internet (…) a bigger change in school mathematics than just counting from another textbook or changing the way of explaining is needed (Nette, adult interview)

As has been illustrated above Nette's interview accounts gradually indicate disaffection with usual school mathematics to an ever increasing extent. They also indicate an emerging awareness of the social constitution of barriers in the participation of students in the classroom. Nette acknowledges that the tediousness she has experienced may be related to situations experienced in an inclusive manner as communicating intelligible ideas and as affording joyfulness in the eyes of other students. Besides being exclusive to some, "the weak ones", she describes school mathematical practices as marginal to "the quiet and conscientious ones" and as inclusive to the communities of "geniuses in mathematics" wherein she as adult definitely doesn't position herself. All the instances where she describes mathematics as in essence a dull/boring/monotonous/useless/meaningless and even painful subject, and increasingly so over the school years, is adding to the firmness of the interpretation. In her comparative accounts both mathematics as a subject and the person Nette appear to end up on the 'wrong sides', where to be absorbed and meaningfully engaged in coming to know within school mathematics, including mathematical problem solving, is a an exception, not a 'normal' state in a person's school mathematical practices.

Final remarks

This paper has illustrated three students' emerging school mathematical identities related to a *learning-by-discovery-of-structur*es-approach in teaching and to *a problem solving perspective* on mathematics teaching and assessment practices mainly materialized in the form of solitary working with mini-problems and project works. Since the stories are situated within practices of school based teacher-research they also shed light on the challenges connected with reform work driven by normative interpretations of theories of knowledge construction. In the case of LOR it can for instance be argued that the use of problem solving activities for assessment of the knowing of individual students worked against the 'learning-by-discovery-of-structures'-approach.

It is the hope of the author that especially the three stories will be helpful for understanding more deeply how the identity work of some engaged and strongly aligned students may emerge into participatory identities, while the identity work of some, many of them girls, emerge into identities of 'not-belonging' to such an extent that they decide to opt out of mathematics already during or immediately after secondary school despite their obvious persistence in and liking of mathematics in general, like Kristina, or of mathematical problem solving and investigative work in particular, like Nette (see e.g. Boaler, 1997a, 2000; Fullerton, 1995, Solomon, 2007).

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